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ORIGINAL PAPER

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Abstract
Electoral systems are rules through which votes translate into seats in parliament. The political economy literature tells us that alternative electoral systems can generate different distributions of power among different social groups in the legislature and therefore lead to different equilibrium economic policies. On the other hand, we know from the endogenous economic growth literature that economic policy can affect growth. What the literature is lacking is a clear link between electoral systems and economic growth. The main objective of this paper is to establish a connection between them. Two main results emerge from our model. First, electoral systems matter for economic growth. Second, the way in which they matter is not straightforward. A precise ranking of these political institutions in terms of economic growth requires the knowledge of the distribution of people among different social classes in society.

Keywords Electoral systems · Party systems · Social classes · Economic growth

JEL Classification O41 · D72 · D78

1 Introduction

Electoral systems map citizens' policy preferences into public policies and public policies affect economic performance. The same preferences under different electoral systems could result in different types of parliaments and therefore, different economic policies and economic outcomes.

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This paper develops an endogenous growth model where electoral systems play an important role in explaining economic outcomes. The economic model is a three-sector (i.e. three-class) dynastic model with limited altruism where the engine of endogenous growth is public investment (*a la* Barro 1990). Our political model makes the choice of public investment endogenous, which is something that previous literature on endogenous growth did not, to our knowledge.

Two types of electoral systems are allowed: a first-past-the-post majoritarian electoral (M) system and a proportional representation (PR) system. Each of these systems will determine, through pre-electoral and parliamentary games, an equilibrium public policy. The equilibrium public policies (rules) will lead to different growth equilibria.

To our knowledge this is the first theoretical attempt to understand how electoral systems affect economic growth. The paper establishes a link between the literature on the effects of different electoral systems on public policy (e.g. Funk and Gathmann 2013; Persson et al. 2007; Persson and Tabellini 2004) and the literature on public policy and growth (e.g. Barro 1990; Barro and Sala-i-Martin 1992).

The main conclusion of this paper is that per se PR and M systems do not necessarily imply different economic growth. This result could explain why some previous works fail to find differences in growth performance across electoral systems (e.g. Persson 2005).

Our model predicts the following ranking in terms of economic growth (from higher to lower): (i) PR in a society with a plurality of the rich class; (ii) M systems and PR in a society with plurality of the middle class; and finally (iii) PR in a society with plurality of the poor class.

In what follows, Sect. 2 briefly reviews the related literature. Section 3 develops the model. Section 4 provides the main results. Section 5 considers an alternative default policy. Finally, in Sect. 6 conclusions are presented.

2 Review of the literature

To our knowledge, there is no single theoretical paper comparing the growth consequences of alternative electoral systems.

Marsiliani and Renström (2007) is the only paper that is relatively close to our aims. In this paper the authors try to analyze the effects on growth of two types of parliamentary democracy under a proportional representation electoral system.

However, the literature on the political economy of growth is extensive. Summaries of the first wave of this literature can be found in Aghion and Howitt (1998, ch.9), Drazen (2000, ch.11) and Persson and Tabellini (2000, ch.14). Acemoglu (2009) devotes the last 2 chapters of his economic growth book to the discussion of the more recent political economy of growth literature.

Much of the early literature explores the effects of income inequality on growth via redistribution. Works along this line include, among others, Perotti (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994) and Glomm and Ravikumar (1992). Reviews of the literature are presented in Benabou (1996), Perotti (1996) and Aghion et al. (1999). However, it also includes models of political instability

(Devereux and Wen 1998) and special interest and rents (Tornell and Velasco 1992; Tornell 1997; Krusell and Rios-Rull 1996).

Any conflict between individuals or classes in this literature is resolved without the mediation of any political system. In most of the papers the assumption of direct democracy, together with majoritarian electoral rule and some version of the median voter theorem are used to determine the political equilibrium (e.g. Alesina and Rodrik 1994; Glomm and Ravikumar 1992; Benabou 1996; Bertola 1993). In others, the "political" equilibrium, is the solution of a game between two or more groups of people, without the mediation of any explicit political institution (e.g. Benabou 1996; Benhabib and Rustichini 1996).

The more recent literature has focused on the role of institutions in economic development and growth. Acemoglu et al. (2005) presents a review of this literature. Most of this literature is empirical or it is not formalized in mathematical models. Some exceptions are the models of Acemoglu and Robinson (2000) and Llavador and Oxoby (2005) on enfranchisement; Persson and Tabellini (2009) and Acemoglu (2008) on the debate democracy vs. non-democracy and economic performance, and Malley et al. (2007) and Economides et al. (2003) on elections, fiscal policy and growth.

There is also a related literature on the consequences of political institutions on economic policy (especially on fiscal policy), that has been developed to an extent (e.g. Persson 2004; Milesi-Feretti et al. 2002; Persson et al. 2000, 2007; Battaglini and Coate 2008; Leblanc et al. 2000). Only more recently this literature has focused attention on the consequences of electoral rules on economic policy (e.g. Ticchi and Vindigni 2010; Iversen and Soskice 2006; Persson and Tabellini 2006).

3 Theory

The assumptions of our model are as follows.

Society We assume a society that is populated by a continuum of dynasties (of mass one). Each dynasty consists of just one individual at a time. Each individual lives for two periods. At the end of their life, an offspring (with the same preferences and technology) takes the place of the parent in the dynasty. Therefore, the dynasty is infinitely lived. These dynasties can be grouped according to their initial level of wealth into three different social classes (poor, middle and rich classes). There is no population growth or overlapping between generations. In this society, for simplicity, each individual is simultaneously a consumer and a producer.

Preferences Individuals care about their consumption of private goods and the bequest (initial wealth) of their children (bequest-as-a-consumption or bequest-as-a-joy-of-giving approach).[1] Among others, this kind of approach has been used by Acemoglu (2009) and Benabou (1996).

[1] Note that this is not the same as to care for their offspring's utility (i.e. altruist motive). See Abel and Warshawsky (1988) for a discussion of the links between joy of giving motive and altruism.

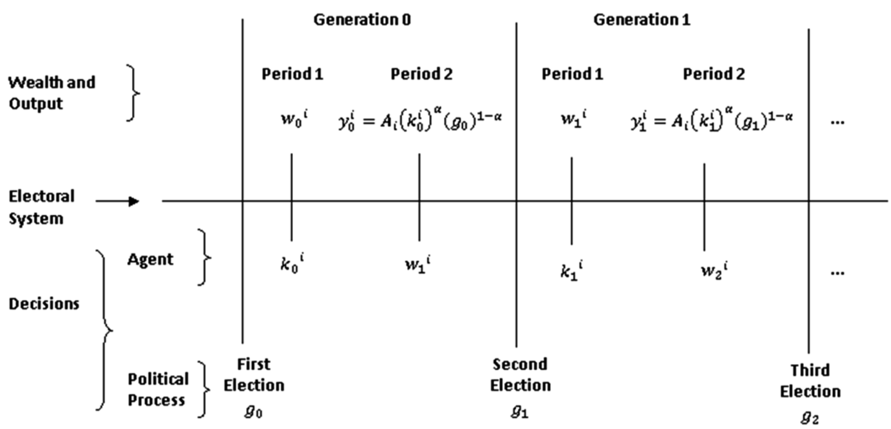

Fig. 1 Timing. Public policy and private decisions

Technology As in Barro (1990), our constant returns to scale production function incorporates two factors of production: private and public capital. The total factor productivity is different across social classes. There is a direct and positive link between initial wealth of the dynasty and its total factor productivity.

Credit and labor market There is no credit market or possibility of transfering money across classes. The only resource available for consumption, investment and tax payments at the beginning of life is the inherited wealth. There is no labor market.

Public policy The government only provides public capital (infrastructure) financing it with a uniform lump-sum tax.

Electoral systems There are two possible electoral systems: a majoritarian electoral system and a proportional representation electoral system. Electoral systems are exogenous and are given at the beginning of history. As we will see later on, each electoral system is associated with a specific type of "party system" (Fig. 1).

The timing of the events in our model is as follows. First, at the beginning of history (generation 0), the electoral system, the initial level of wealth and total factor productivity of each dynasty are given. Second, each individual votes once at the beginning of their life. The votes and the electoral system will determine a particular configuration of power in parliament. Third, public policy is the result of a bargaining process among different groups in parliament. Fourth, once public policy is implemented, each citizen decides in the first period of their life how much to invest and in the second period how much they will bequeath (i.e. the initial level of wealth of the next generation of the dynasty). Production takes place in the second period of life. For generation t (for $t > 0$) the timing of events is exactly as before, the only difference is that now the initial wealth is inherited from the previous generation.

Note that there is an asymmetry between the assumption that it is not possible to transfer savings from period 1 to 2 and the assumption that it is possible to transfer wealth from generation t to generation $t+1$. However, this can be easily avoided (without any changes in the results) assuming instead that in the second period the decision is about how much to spend in education and not how much wealth to leave to the offspring. With this assumption and the additional assumptions that the production function of "education" is linear in wealth, and that the level of education of the offspring enters into the utility function instead of their initial wealth, we will arrive to identical results. For the sake of simplicity, we will keep this "asymmetric assumption" about the possibility to transfer resources within and between generations.

3.1 The economic model

3.1.1 Assumptions

The society is comprised of three different classes: p, m and r (poor, middle, and rich class) with size s^p, s^m, s^r respectively, (for simplicity assume $\sum_{j=p,m,r} s^j = 1$). We also assume that max $\{s^p, s^m, s^r\} < \frac{1}{2}$. With this assumption we avoid having the "uninteresting" case of a "natural majority" in the society. Note that the previous assumption implies that $s^i + s^j > \frac{1}{2}\ \forall i \neq j\,(i, j = p, m, r)$, (i.e. the number of people in any two classes is more than a half of the total population).

The initial level of wealth only differs across social classes. By definition (of social classes) we have that $w_0^p < w_0^m < w_0^r$, where w_0^i is the initial wealth of dynasties belonging to class $i\,(i = p, m, r)$.

In our model, individuals are simultaneously consumers and producers.

The utility function of an individual belonging to class i and generation t is

$$u_t^i = \log(c_{1,t}^i) + \rho\gamma \log(c_{2,t}^i) + \rho(1-\gamma)\log(w_{t+1}^i), \tag{1}$$

where $c_{j,t}^i$ is the private consumption of an individual belonging to class i and generation t in period $j = 1, 2$, w_{t+1}^i is the bequest that generation t gives to generation $t+1$ (or the initial level of wealth of generation $t+1$), ρ ($\rho < 1$) is a discount factor and γ $(0 < \gamma < 1)$ is a parameter that measures the relative importance of consumption and the bequest in the utility function.

The production function is class specific and is given by the following Cobb–Douglas technology

$$y_t^i = A_i (k_t^i)^\alpha (g_t)^{1-\alpha}, \tag{2}$$

where k_t^i is the private capital[2] of an individual belonging to class i and generation t, g_t is the public capital (e.g. infrastructure) in per capita terms,[3] A_i is a class specific total factor productivity parameter, α (with $0 < \alpha < 1$) is a parameter. In our model capital and investment are synonyms, since we are assuming, for simplicity,

[2] Probably the best way of interpret it is as a composite index of physical and human capital.

[3] Implicitly we are assuming some kind of congestion. As Barro and Sala-i-Martin (1992) points out most of public goods and services suffer from some kind of congestion, and this is typically the case with roads and education. However, note that because there is no population growth in our model, this is not an important assumption.

complete depreciation in one generation. Production takes place at the beginning of period 2.

We are assuming that the total factor productivity A depends positively on the initial level of wealth of dynasties, i.e. $A_p < A_m < A_r$, and is inherited by the following generations. The first assumption can be interpreted as ability differentials across dynasties. There is some evidence that supports the assumption of different total productivity across social classes. For example Duflo (2006) and Banerjee and Duflo (2007, 2008) show that poor families have significantly lower productivity than other families in some poor countries. Many reasons can explain this fact, credit constraints, inexistence of insurance markets, land tenancy arrangements, small scales of production, not enough intake of calories, etc. With respect to the second assumption, Black et al. (2005) presents evidence suggesting that ability can be inherited.

In the first period of their life, after w_t^i is inherited, consumer-producers decide how much to invest, k_t^i, and they pay lump-sum taxes T_t ($T_t \geq 0$). In period 2, after production is realized, they decide the level of bequest for the next generation, w_{t+1}^i. Under these assumptions, the budget constraints of an individual belonging to class i and generation t are

$$c_{1,t}^i = w_t^i - T_t - k_t^i, \tag{3}$$

$$c_{2,t}^i = y_t^i - w_{t+1}^i. \tag{4}$$

We are assuming an every-period balanced public budget, i.e.:

$$g_t = T_t. \tag{5}$$

There is no credit or labor market in this economy. Therefore, bequests are indispensable for the propagation of the dynasty in our model.

3.1.2 Policy

Each individual can directly choose the level of investment and bequest and indirectly (voting) the fiscal policy. Let us first find the optimal investment and bequest functions for a generic individual.

Taking into account Eqs. (1)–(5) the problem for an individual belonging to class i and generation t is

$$\max_{k_t^i, w_{t+1}^i} u_t^i = \log(w_t^i - g_t - k_t^i) + \rho\gamma \log\left[A_i (k_t^i)^\alpha (g_t)^{1-\alpha} - w_{t+1}^i\right] + \rho(1-\gamma)\log(w_{t+1}^i), \tag{6}$$

From the first order conditions we have that optimal investment and bequest functions:

$$k_t^i = \frac{\rho\alpha}{1+\rho\alpha}(w_t^i - g_t) \text{ for } g_t \leq w_t^i, \text{ 0 otherwise}, \tag{7}$$

$$w^i_{t+1} = (1-\gamma)y^i_t. \tag{8}$$

It can be verified that the second order condition for a maximum is in place for this problem.

As a result of the assumption of non-existence of credit markets, the level of wealth of individual i is key to determine the optimal level of capital and the bequest.

From (7), (8) and (2) we obtain

$$y^i_t = A_i\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}(w^i_t - g_t)^{\alpha}(g_t)^{1-\alpha}, \tag{9}$$

$$w^i_{t+1} = (1-\gamma)A_i\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}(w^i_t - g_t)^{\alpha}(g_t)^{1-\alpha}. \tag{10}$$

With these results, the indirect utility function, v^i_t, of an agent belonging to class i and generation t, for $w^i_t > g_t > 0$, can be written as

$$v^i_t(w^i_t, g_t) = D_i + (1+\rho\alpha)\log(w^i_t - g_t) + \rho(1-\alpha)\log(g_t), \tag{11}$$

where $D_i \equiv \log\left\{\left[A_i\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}\right]^{\rho}\left(\frac{1}{1+\rho\alpha}\right)\right\}$.

At $g_t = 0$, $v^i_t(w^i_t, g_t = 0) = \log(w^i_t)(= u^i_t(w^i_t, g_t = 0))$.

If we maximize (11) with respect to g_t we obtain the preferred public policy of a citizen belonging to class i and generation t:[4]

$$g^i_t = \frac{\rho(1-\alpha)}{1+\rho}w^i_t. \tag{12}$$

This implies $g^p_t < g^m_t < g^r_t$ as long as $w^p_t < w^m_t < w^r_t$.

Note that the level of wealth determines the desired level of public capital. This implies that different social classes could eventually vote for different public policies (or parties). Richer classes will prefer higher public investment.

The intuition behind this result is the following. Richer classes want to transfer more resources to period 2 of their life, but for this they need to produce more output and because private and public capital are complementary, they want a higher level of both. This implies in particular that they prefer a higher level of public investment.[5]

3.2 The politico-institutional model

In our model there are only two possible electoral systems: a majoritarian system and a proportional representation system. The most accepted way of characterizing

[4] Again, it can be verified that the second order condition for a maximum is in place.

[5] Remember that production is the only way of transferring resources from one period to the other.

these systems is via the electoral formula associated to them, or in other words, how votes are counted to distribute seats.[6]

The M system is characterized by the first-past-the-post principle, the winner takes all the seats of the relevant electoral district. Under a PR system seats are distributed according to the proportion of votes obtained by each candidate/party in the relevant electoral district.

Norris (2004), in chapter 2 of her book *Electoral Engineering: Voting Rules and Political Behavior*, summarizes the main characteristics of these two electoral systems.

"The aim of majoritarian electoral systems is to create a 'natural' or a 'manufactured' majority, that is, to produce an effective one-party government with a working parliamentary majority while simultaneously penalizing minor parties, especially those with spatially dispersed support. In 'winner take all' elections, the leading party boosts its legislative base, while the trailing parties get meager rewards."

"[P]roportional representation electoral systems focus on the inclusion of all voices, emphasizing the need for and bargaining and compromise within parliament, government, and the policymaking process. The basic principle of proportional representation (PR) is that parliamentary seats are allocated according to the proportion of votes cast for each party."[7]

Thus, majoritarian electoral systems tend to generate an overrepresentation in parliament of the party with most votes in the election, (more seats in parliament than votes in the election), while PR systems generate a distribution of seats in parliament that it is closer to the proportion of votes obtained by each party in the election.

Electoral systems have important implications in terms of party systems' structure. In this paper we will make the party system endogenous (as in Ticchi and Vindigni 2010).

As we will see, under a majoritarian electoral system we will only have policy-oriented candidates that belong to the middle class (the number of candidates will remain undetermined). Under a majoritarian electoral system only the middle class will have seats in parliament.

Candidates from the three classes will be participating in the electoral process under a PR system (but again, how many of them will be remain undetermined). Under a PR system, the proportion of seats in parliament of class i will be s^i (for $i = p, m, r$).

Let us now assume that the number of candidates is endogenous (similarly to Osborne and Slivinsky 1996 and Besley and Coate 1997). Each voter

[6] Of course we can classify electoral systems according to a vector of characteristics; but as Norris (1997, p. 299) points out even when we can include for this classification "district magnitude, ballot structures, effective thresholds, malapportionment, assembly size, and open/closed lists, ...the most important variations concern electoral formulas".

[7] Colomer (2004, p. 10) also describes the characteristics of these systems along similar lines:

"...electoral systems based on the majority principle, ...tend to produce a single, absolute winner and subsequent absolute losers, ...proportional representation, [is] a principle forged to create multiple partial winners and much fewer losers than majority rule."

(consumer–producer) can choose to become a candidate at the election. By participating as a candidate, she incurs in a utility loss of C (leisure loss) and if she wins the election, she obtains an extra utility of Z (ego rents), where $Z > C$. A citizen runs for office if and only if the expected return of doing so is greater than the associated costs.

The game has three stages: (1) entry of candidates stage: each citizen decides whether or not to become a candidate (knowing s^i $\forall i$); (2) election stage: the members of the parliament are elected in a single nation-wide electoral district where every citizen has the right to vote; (3) parliamentary stage: at least one-half of the parliament must approve the policy to be implemented.

3.2.1 Majoritarian electoral system

We will use backward induction to find the political equilibrium.

Parliamentary stage Note that because there is only one electoral district and the winner takes all the seats, the government is formed with only one class, say class i, and the policy to be implemented is its preferred one, i.e. $g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i$.

Election stage Assuming sincere voting,[8] the voter j will vote for a candidate $f \in \Omega$ (the set of candidates) if f is such that $v_t^j(g_t^f, w_t^j) > v_t^j(g_t^i, w_t^j)$ $\forall i \in \Omega$, where g_t^f represents the optimal policy for candidate f and g_t^i for candidate i. Of course this condition implies that if a candidate of the same class of voter j is available, then the vote of j goes to this candidate. Otherwise, the vote goes to the candidate that maximizes j's utility given that a candidate of his class is not available. Note that voter j could be indifferent between two candidates if their optimal policy is the same, i.e. if they are from the same class. If this is the case, we assume that every candidate of the same class has the same probability of receiving the vote.

Entry of candidates stage First, note that in the election, potentially, can be candidates of only one class, two classes or even three different classes. What we will prove is that the model has a unique equilibrium where only candidates belonging to the middle class (those who prefer the median policy) will participate in the election. First, note that because in our model the median voter theorem applies, in any pairwise vote the median policy (and the m class candidate) will win. Then candidates belonging to the other classes (p, r) will not participate if a candidate of the m class is participating, and in this way they will avoid incurring in a net cost of C. Next we will prove that having an election with candidates belonging to the three classes is not an equilibrium. Note that if candidates of all three classes participate, because one of the classes has plurality, this class will win the election with certainty, then the candidates belonging to the other classes will not participate to avoid paying C. Thus, the only possible equilibrium is the one with only candidates of the m class.

[8] This assumption implies that each citizen votes for the policy (or candidate) that brings him/her the maximum utility, ignoring the possible effects that his/her decision and those of others could have on the election outcome. We can justify the sincere voting assumption by noticing that we have a continuum of (or infinite) agents in our model and therefore, the probability of an agent being pivotal tends to zero. Then voters vote for their first best option without any strategic consideration.

Second, we need to prove that the set of candidates is not empty. Let q^m be the probability of victory for a particular middle class candidate (in a symmetric equilibrium q^m will be the same across candidates of this class). A middle class candidate will be running for office if her expected gain exceed the expected cost. Then, to prove that the set of candidates is not empty, it is enough to prove that this net gain is positive when there is only one candidate. If there is only one candidate $q = 1$ and her participation constraint can be written as $\left[v_t^m(w_t^m, g_t^m) - v_t^m(w_t^m, g_t^i)\right] + Z - C \geq 0$ (the term in square brackets represents the gain of implementing her preferred policy). Since g_t^m maximizes v_t^m (by definition) the expression in square brackets is always non-negative and because $Z > C$, then $\left[v_t^m(w_t^m, g_t^m) - v_t^m(w_t^m, g_t^i)\right] + Z - C > 0$. Of course it could exist more than one middle class candidate, and because there is free entry of candidates, in general there will be as many middle class candidates as needed to make the expected net gain of participating in the election equal to zero.

3.2.2 Proportional representation electoral system

The assumption of sincere voting implies that if a candidate of our class is available we will vote for her. This assumption together with the assumption of a PR system implies that there is the probability of candidates from the three different social classes (they all have now a positive probability of being elected).[9]

Additionally, if we assume that the parliament is large enough as that a single additional seat for any of the parties does not affect the policy outcome, then the only variables that matter at the time of deciding participation are Z, C and the endogenous probability of being elected, q^i. A candidate will run for office if $q^iZ > C$. In equilibrium (if there is perfect competition) we will have enough candidates of each class as to make $q^i = C/Z$, and each class will win exactly s^i seats (more formally we are assuming that the parliament is composed by a continuum of legislators of mass δ, where $0 < \delta < 1$).[10]

4 Politico-economic equilibrium

The policy formation in our model is an outcome of a process of bargaining (bargaining game) between political parties in parliament. We will assume that to pass legislation it is necessary to achieve the majority of votes in parliament. The protocol of bargaining is very simple. The representative (randomly appointed) of the party with a plurality in parliament, say party i, put forward a policy proposal, g_t^{ij}, to

[9] Note that each class under PR can can win up to s^i seats of the parliament.

[10] Note that the number of candidates of class i will be greater than $s^i\delta$. $s^i\delta$ candidates will be elected with probability $q^i = 1$, and this will imply that each candidate will get an expected gain of $Z - C > 0$. However, if there is free entry and perfect competition, new candidates will arrive until $q^iZ - C = 0$ in which case the number of candidates must be greater than $s^i\delta$. This implies that there are enough candidates as to elect $s^i\delta$.

the head of one other party, say party j.[11] If the proposal is accepted, a coalition is formed and the agreed proposal is implemented. If the proposal is rejected a default policy is implemented: $g_t = 0$. As in Besley and Coate (1998) public investments satisfying standard criteria of efficiency will not necessarily be adopted at political equilibrium.

A politico-economic equilibrium simultaneously involves two types of equilibrium: a political equilibrium and an economic equilibrium. These equilibria are defined as follows.

Definition 1 (*Political equilibrium or equilibrium public policy*) An equilibrium public policy is a policy that is the equilibrium outcome of the bargaining game in parliament.

Definition 2 (*Economic equilibrium or balanced growth equilibrium*) A balanced growth equilibrium is characterized by a pair of constant relative levels of wealth, $\left[\left(\frac{w^p}{w^m}\right)^*, \left(\frac{w^r}{w^m}\right)^*\right]$, such that $\frac{w^i_{t+1}}{w^i_t} = \frac{y^i_{t+1}}{y^i_t} = \frac{k^i_{t+1}}{k^i_t} = \frac{g_{t+1}}{g_t} = \mu \, \forall i \, (i = p, m, r)$, (i.e. such that all the variables in the economy are growing at the same constant rate, $\mu - 1$).

4.1 Majoritarian electoral system

Only middle class politicians are in parliament at $t = 0$ and g^m_0 will be the chosen policy

The problem is that for generations $t > 0$, w^i_t is endogenous, so *a priori* we do not know if the so called middle class at time 0 will still be the middle class in the future and thus we cannot be sure that the median voter belongs to this class for all $t > 0$.

However, it can be proved that there is no social mobility in our model (i.e. $w^p_t < w^m_t < w^r_t \, \forall t$; see Appendix A), in which case, the median voter is always an individual belonging to class m.

The intuition behind the no social mobility result is simple. Richer classes are more productive and subsequently they always produce more output for the same level of g_t, but because all classes have the same utility function, they will leave the same proportion of their output for the next generation [see (8)]. Therefore, richer classes will leave more bequest in absolute terms. However, this bequest is nothing else than the next period wealth. So, the relative position of each class in

[11] The party that has plurality in parliament is usually the party that is in charge of executive power and usually has a prerogative over budget requests. Therefore it is reasonable to assume that the party that has plurality is the one with agenda-setting power. Posner and Park (2007, p. 5–6) discussing trends in budgeting point out that: "Legislatures themselves delegated powers to the executive, wary of their own instincts to favour particular constituency-based policies at the expense of the broader fiscal wellbeing of the country. Moreover, legislatures did not have expertise to keep up with the growing sophistication and complexity of modern budgets, particularly when compared to the detailed knowledge possessed by executive bureaucracies (Schick, 2002).... The eclipse of the legislative role in budget formulation was reflected in the limited formal roles legislatures were given in developing and approving budgets. Legislatures had little formal power to review or approve overarching budget targets or policies, nor were legislatures generally involved in approving medium-term expenditure frameworks."

the society is preserved over time. In other words, there is no social mobility in our model.

With these results we can state our first proposition.

Proposition 1 *Under a majoritarian electoral system the model has a single political equilibrium. Only politicians of type m are in parliament and their preferred policy is implemented:* $g_t = g_t^m = \frac{\rho(1-\alpha)}{1+\rho} w_t^m \; \forall t$.

Now, let us proceed with finding the balanced growth equilibria of our model. From (10) and the equilibrium fiscal policy (Proposition 1) we have that

$$\frac{w_{t+1}^m}{w_t^m} = A_m(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}. \tag{13}$$

The wealth of the middle class grows at a constant rate. Therefore, if an equilibrium exist it will necessarily imply $\mu = \mu_m \equiv A_m(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$. We will assume that the parameters are such that $\mu_m > 1$ (positive growth rate).

For the other classes we have that, again using (10) and the equilibrium fiscal policy,

$$\frac{w_{t+1}^i}{w_t^i} = cA_i\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w_t^m}{w_t^i}\right)^\alpha \left(\frac{w_t^m}{w_t^i}\right)^{1-\alpha}, \tag{14}$$

where $c \equiv (1-\gamma)\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^\alpha\left(\frac{\rho(1-\alpha)}{1+\rho}\right)^{1-\alpha}$.

Imposing the balanced growth equilibrium condition $\frac{w_{t+1}^i}{w_t^i} = \mu_m$ to (14), and solving for $\frac{w_t^m}{w_t^i}$, we can find the equilibrium relative (to middle class) level of wealth for an individual of class i. The existence of this relative wealth is sufficient to ensure that the growth rate of wealth and of other variables is the same across social classes, i.e. that an equilibrium exist.

The following conditions will be sufficient for the existence of equilibria (Condition 1) and for convergence (Condition 2) (see Appendix B).

Condition 1 $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^\alpha$.

It implies that the total factor productivity parameter A_m must relatively close to A_p.

Condition 2 $x^1 + \frac{\rho(1-\alpha)}{1+\rho} \leq \frac{w_0^p}{w_0^m}$.

x_1 and x_2, with $x_1 \leq x_2$, are the solutions to the equation $x = -\frac{\rho(1-\alpha)}{1+\rho} + \frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^\alpha (x)^\alpha$.

The condition implies a restriction on the initial distribution of wealth: w_0^p must be relatively close to w_0^m.

Finally, given that $\frac{w^i_{t+1}}{w^i_t} = \mu_m \ \forall i$, it is easy to show that all the other variables of the economy will be growing at the same rate (see Appendix B).

Proposition 2 *Under a majoritarian electoral system and Condition 1, i. there exist two balanced growth equilibria,*[12] *and in both equilibria all the variables of the economy grow at rate* $(\mu_m - 1)$. *ii. If Condition 2 is also satisfied the economy converge to an equilibrium.*

Proof Appendix B. □

4.2 Proportional representation electoral system

First note that the distribution of seats in parliament maps perfectly the distribution of people among classes in society: s^p, s^m, s^r.

Let us now analyze the parliamentary game and the economic equilibria.

Because a priori we do not want to impose further restrictions on the distribution of people among social classes, we will find the equilibrium under three different alternatives (we are ruling out the possibility of equal-size groups): (i) $\max\{s^p, s^r\} < s^m$ (ii) $\max\{s^m, s^r\} < s^p$ (iii) $\max\{s^m, s^p\} < s^r$.

Before proceeding to analyze each of these cases, let us discuss first the conditions under which a proposal is accepted in the parliamentary game.

Note that when class (party) i receives the offer of forming a coalition with class (party) j, it will accept it as long as the proposed policy g^{ij}_t gives it an utility greater than the default policy $g_t = 0$. The region of acceptance is defined by $g_t \in \left[\underline{g}^i_t, \bar{g}^i_t\right]$, where $\underline{g}^i_t$, $\bar{g}^i_t$, with $\underline{g}^i_t < \bar{g}^i_t$, are such that the default utility (i.e. $u^i_t(w^i_t, g_t = 0)$) is equal to the indirect utility evaluated at these points, i.e. $\underline{g}^i_t$, $\bar{g}^i_t$ are defined by $\log(w^i_t) = v^i_t(w^i_t, g_t)$ (note that $u^i_t(w^i_t, g_t = 0) = \log(w^i_t)$). For g_t in the interval $\left[\underline{g}^i_t, \bar{g}^i_t\right]$ the utility is equal or greater than the default utility (because the indirect utility is concave).

Now note that if $u^i_t(w^i_t, g_t = 0) = v^i_t(w^i_t, g_t = 0)$, then $\underline{g}^i_t = 0 \ \forall i$. Even though it is not possible to analytically determine $\bar{g}^i_t$, we can say more about it. Note that from the concavity of the indirect utility function $0 = \underline{g}^i_t < g^i_t < \bar{g}^i_t$, where $g^i_t = \arg\max_{g_t} v^i_t(w^i_t, g_t)$. Additionally, $\bar{g}^p_t < \bar{g}^m_t < \bar{g}^r_t$ (see Appendix D).

The middle class has plurality of votes ($\max\{s^p, s^r\} < s^m$*) in parliament* Note that because $\bar{g}^m_t < \bar{g}^r_t$ and $0 < g^m_t < \bar{g}^m_t$, then $0 < g^m_t < \bar{g}^r_t$ and therefore $g^m_t \in \left[\underline{g}^r_t = 0, \bar{g}^r_t\right]$.

[12] These 2 equilibria are defined by 2 different $\frac{w^p_t}{w^m_t}$ ratios. The equilibrium defined by the smallest ratio is unstable, while the other is stable.

Given the last result, class m will be able to choose its first best policy, g_t^m, since this policy is better for r than the default policy. So, there is always at least one party (party r) that will accept g_t^m.

The poor class has plurality of votes ($\max\{s^m, s^r\} < s^p$*) in parliament* Following the same reasoning that in previous case, p will be able to choose its first best policy g_t^p. For both, m and r, g_t^p is better than the default policy.

The rich class has plurality of votes ($\max\{s^m, s^p\} < s^r$*) in parliament.* Without having prior knowledge of the parameters and initial distribution of wealth of the model, we cannot know if they will be able to find a partner for their first best policy. However, if the initial distribution of wealth is such that $g_0^r < g_0^{\bar{m}}$, r will be able to find a coalition partner for its first best policy. In this case we know that at least for the middle class g_0^r is better than the default policy.

Proposition 3 summarizes previous results.

Proposition 3 *Under a PR electoral system where alternatively (i)* $\max\{s^p, s^r\} < s^m$, *(ii)* $\max\{s^m, s^r\} < s^p$, *or (iii)* $\max\{s^m, s^p\} < s^r$ *and* $g_0^r < g_0^{\bar{m}}$ *the political equilibrium implies, respectively, (i)* $g_t = g_t^m = \frac{\rho(1-\alpha)}{1+\rho} w_t^m \; \forall t$, *(ii)* $g_t = g_t^p = \frac{\rho(1-\alpha)}{1+\rho} w_t^p \; \forall t$, *(iii)* $g_t = g_t^r = \frac{\rho(1-\alpha)}{1+\rho} w_t^r \; \forall t$.

Proposition 4 *Under a PR electoral system where* $\max\{s^p, s^r\} < s^m$ *and Condition 1 (i) two balanced growth equilibria exist,* [13] *in both equilibria all the variables of the economy grow at rate* $(\mu_m - 1)$ *with* $\mu_m \equiv A_m(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$. *(ii) if additionally Condition 2 is verified the economy will converge to an equilibrium.*

Proof Appendix B. □

In terms of the evolution of the economy, we are exactly in the same case as under a M system. Then, if the middle class has a plurality of votes, PR and M are equivalent in terms of economic outcomes. In other words, when the middle class has plurality of votes, the electoral system is irrelevant for growth.

Proposition 5 *Under a PR electoral system where* $\max\{s^m, s^r\} < s^p$, *(i) exists a single balanced growth equilibrium where all variables in the economy grow at rate* $(\mu_p - 1)$ *with* $\mu_p \equiv A_p(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$; *and (ii) there is always convergence to this equilibrium.*

Proof Appendix E. □

Note that from (10) and (12) for p we have

[13] These 2 equilibria are defined by 2 different $\frac{w_t^p}{w_t^m}$ ratios. The equilibrium defined by the smallest ratio is unstable, while the other is stable.

$$\frac{w^p_{t+1}}{w^p_t} = A_p(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}, \tag{15}$$

and thus if an equilibrium exist (and it can be proved that it always exist, see Appendix E) it will imply necessarily that the economy will be growing at rate $(\mu_p - 1)$, where $\mu_p \equiv A_p(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$.

For the other classes we have that

$$\frac{w^i_{t+1}}{w^i_t} = cA_i\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w^p_t}{w^i_t}\right)^\alpha\left(\frac{w^p_t}{w^i_t}\right)^{1-\alpha}. \tag{16}$$

It can be shown that there exists $\frac{w^p_t}{w^i_t}$ (for $i =, m, r$) such that $\frac{w^i_{t+1}}{w^i_t} = \mu_p\ \forall i$, i.e. an equilibrium exist (see Appendix E). Again, given this result, it can be shown that all the other variables in the economy are also growing at rate $(\mu_p - 1)$. Additionally, it also can be shown that there is always convergence to the equilibrium.

Let us introduce two additional conditions necessary for existence of equilibrium and convergence under a PR electoral system where r has plurality.

Condition 3 $\frac{A_r}{A_p} \le \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^\alpha$.

This condition implies that the levels of productivity of r and p are relatively close to each other.

Condition 4 $x'_1 + d \le \frac{w^p_0}{w^r_0}$ and $x''_1 + d \le \frac{w^m_0}{w^r_0}$.[14]

x'_1 and x'_2, with $x'_1 \le x'_2$, are now the solutions to the equation $x' = -d + b_{pr}\left(x'\right)^\alpha$, where $b_{pr} \equiv \frac{A_p}{A_r}\left(\frac{1+\rho}{1+\rho\alpha}\right)^\alpha$; and x''_1 and x''_2, with $x''_1 \le x''_2$, are the solutions to the equation $x'' = -d + b_{mr}\left(x''\right)^\alpha$, where $b_{mr} \equiv \frac{A_m}{A_r}\left(\frac{1+\rho}{1+\rho\alpha}\right)^\alpha$. Condition 4 is necessary for convergence to the stable equilibrium.

Proposition 6 *Under a PR electoral system where r has plurality,* $g^r_0 < \bar{g}^m_0$*, and Condition 3 is satisfied, (i) up to four balanced growth equilibria exist*[15] *in all of them all the variables of the economy grow at rate* $(\mu_r - 1)$*. (ii) If Condition 4 is also satisfied the economy will converge to the stable equilibrium* $(\mu_r - 1)$ *with* $\mu_r \equiv A_r(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$.

Proof Appendix F. □

[14] Conditions 2 and 5 are enough to avoid the disappearance of a class (i.e. the case where $T_t \ge w^i_t$ for some i).

[15] If condition 3 is verified with strict inequality there will be 4 equilibria. These 4 equilibria are defined by the 4 possible combinations of 2 equilibrium values of $\frac{w^p_t}{w^r_t}$ and 2 equilibrium values of $\frac{w^m_t}{w^r_t}$. Only the equilibrium defined by the maximum value of both ratios is stable.

Table 1 Political institutions and economic growth

Electoral system	Growth (μ)[a]	Public investment (g_t)[b]
Majoritarian	$A_m E$	Fw_t^m
Proportional representation		
(i) $\max\{s^p, s^r\} < s^m$	$A_m E$	Fw_t^m
(ii) $\max\{s^m, s^r\} < s^p$	$A_p E$	Fw_t^p
(iii) $\max\{s^m, s^p\} < s^r$ and $g_0^r < \bar{g}_0^m$	$A_r E$	Fw_t^r

[a] $E \equiv (1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$

[b] $F \equiv \frac{\rho(1-\alpha)}{1+\rho}$

If alternatively $g_0^r > \bar{g}_0^m$, then we have that the restrictions are active (i.e. $\bar{g}_0^p, \bar{g}_0^m < g_0^r$). In this case, because $\bar{g}_0^p < \bar{g}_0^m < g_0^r$ and v^r is concave (i.e. because both $\bar{g}_0^p, \bar{g}_0^m$ are at the left of the maximum, and $v^r(\bar{g}_0^p) < v^r(\bar{g}_0^m)$), r will choose m as coalition partner (the cheapest class to buy as coalition partner) and will offer them $g_t^{rm} = \bar{g}_t^m > g_t^m$, the minimum utility for participation, and the offer will be accepted. Unfortunately, in this case it is not possible to find a closed form solution.

4.3 Growth comparisons across electoral systems

It is important to note that per se the PR and the M systems do not necessarily imply different economic growth. For example, if the middle class has plurality in the society both systems will generate a growth rate of $A_m E$ (where $E \equiv (1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$). In order to produce a precise ranking, we need to know the distribution of people among classes. This could be one of reasons why empirical works (e.g. Persson 2005) fail to find a clear link between electoral systems and growth. In other words, we need to take into account the distribution of people among classes before making any prediction of the impact of the electoral system on growth.[16]

The raking in terms of economic growth and public investment (from higher to lower) is:

1. PR in societies with plurality of the rich class, $A_r E$,
2. M systems and PR in societies with plurality of the middle class, $A_m E$, and finally
3. PR in societies with plurality of the poor class, $A_p E$ (Table 1).

[16] It is important to notice that in our model higher growth does not imply necessarily higher welfare, so our theory should not be interpreted as an argument in favor of a pro-rich class government. Distributional considerations are important for welfare. It is easy to show that under an utilitarian social planner the growth will be: $\mu - 1 = A_r\left[s^p\left(\frac{A_p}{A_r}\right)^{1/(1-\alpha)} + s^m\left(\frac{A_m}{A_r}\right)^{1/(1-\alpha)} + s^r\right]^{1-\alpha} E - 1.$

5 Extension: an alternative default policy

Let us assume now a more realistic default policy. We will now assume that if an agreement is not reached in parliament the public investment is maintained at the level of the previous period, i.e. the default policy is $g_t = g_{t-1}$, and for period 0 the default policy is $g_0 = 0$.

Under the majoritarian electoral system, the middle class can freely choose its best policy. Subsequently, results under this electoral system are the same as before.

The results under a PR system are also exactly the same as before when the middle class or the poor class have plurality (see Appendix G). The intuition is the following. Class r always prefers a public investment that it is bigger than that which is optimal for classes p and m. For period 1 the default policy is 0, therefore r will accept the offer of class i in period 1. Now, in period two, (because we are assuming that the parameters of the model are such that there is economic growth), the optimal public investment for class i is bigger than its preferred (the equilibrium policy) in period 1. Therefore, we will have the following situation in period 2: $g_1 = g_1^i < g_2^i < g_2^r$. In this case, for class r, is better to accept g_2^i than to have the default policy $g_1 = g_1^i$. By complete induction it can be proved that this is also valid for any $t > 2$.

Therefore, the optimal policy of class i (for $i = p, m$) is an equilibrium public policy, and from here the same results of Sect. 4.2 follow.

The case of PR with r having plurality is more complex now. For the first period the public policy (offer of r) will be such that makes m indifferent between accepting or rejecting it. This could imply $g_0 = g_0^r$ or $g_0 = g_0^m$ depending on the parameters' values, as in Sect. 4.2. The difference now is that even if $g_0 = g_0^r$, there is no guarantee that in the next periods we will continue to have $g_t = g_t^r$. The only thing that can be proved is that $g_t^m \leq g_t \leq g_t^r$. Note that $g_t < g_t^m$ cannot be an equilibrium because both are better off increasing g_t at least up to g_t^m, and $g_t > g_t^r$ cannot be an equilibrium because again both will be better off reducing g_t to the level g_t^r. With the functional forms that we have, it is not possible to find an analytical solution, but how close will be g_t of g_t^m will depend on how close is g_{t-1} of g_t^m. In other words, as closer the default policy gets to the optimal policy of m, more will need r to offer m to find a coalition partner.

6 Conclusions

Do different electoral systems deliver different economic growth? The distribution of people among classes seems to be the key factor to be taken into account before answering this question.

This is not surprising. After all, political institutions are means of "aggregating" preferences and as such, they could be biased towards different classes and therefore deliver different policies and economic outcomes under different social structures.

Our model generates the following ranking in terms of economic growth: (i) PR in societies with plurality of the rich class; (ii) PR in societies with plurality of the middle class and M systems; and (iii) PR in societies with plurality of the poor class.

Our model represents only the first step towards comprehending how electoral systems affect economic growth. As such, there are many possible ways forward.

One limitation of our model is the result of no mobility of social classes, it would be interesting to include social mobility in the model. This could advance our knowledge of how social dynamics interacting with institutions affect economic growth.

Another interesting line of research is to explore a model with alternative types of taxes.

The form of government (e.g. parliamentary vs presidential) it is also important for policy, therefore, we expect it to influence economic policy and growth. The analysis of the links between form of government and economic growth is therefore a very relevant line for future research.

Finally, it could be interesting to analyze a more realistic and complex bargaining process in parliament (for example the Baron–Ferejhon legislative bargaining) and its implications for growth.

No social mobility property

We have to prove that $w_t^p < w_t^m < w_t^r\ \forall t$. This is true for period 0 by assumption. By complete induction we will prove that this is also true for any $t > 0$.

First, let us prove that if $w_0^p < w_0^m < w_0^r$ then $w_1^p < w_1^m < w_1^r$. From (10) we have that $\frac{w_{t+1}^i}{w_{t+1}^m} = \frac{A_i}{A_m}\left(\frac{w_t^i - g_t}{w_t^m - g_t}\right)^\alpha$, then $\frac{w_1^i}{w_1^m} = \frac{A_i}{A_m}\left(\frac{w_0^i - g_0}{w_0^m - g_0}\right)^\alpha$. Since $A_p < A_m < A_r$ and $w_0^p < w_0^m < w_0^r$, then $\frac{w_1^r}{w_1^m} = \frac{A_r}{A_m}\left(\frac{w_0^r - g_0}{w_0^m - g_0}\right)^\alpha > 1$ and $\frac{w_1^p}{w_1^m} = \frac{A_p}{A_m}\left(\frac{w_0^p - g_0}{w_0^m - g_0}\right)^\alpha < 1$. Therefore $w_1^p < w_1^m < w_1^r$.

Second, following the same steps as before it can be proved that if $w_{t-1}^p < w_{t-1}^m < w_{t-1}^r$, given that $A_p < A_m < A_r$, then $w_t^p < w_t^m < w_t^r$.

Then by complete induction, $w_t^p < w_t^m < w_t^r\ \forall t$.

Propositions 2 and 4

Existence of a steady-growth equilibrium

First note that because [from (13)] w_t^m is always growing at rate $\mu_m - 1$, if the ratio $\frac{w_t^m}{w_t^i}$ is constant then w_t^i must also be growing at rate $\mu_m - 1$. Using this argument we will prove that under some conditions an equilibrium exist.

From (13) and (14) we have that

$$\frac{\frac{w^i_{t+1}}{w^i_t}}{\frac{w^m_{t+1}}{w^m_t}} = \frac{cA_i}{\mu_m}\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w^m_t}{w^i_t}\right)^{\alpha}\left(\frac{w^m_t}{w^i_t}\right)^{1-\alpha}, \tag{17}$$

and this expression can be rewritten as

$$x^i_{t+1} = -d + b_{im}\left(x^i_t\right)^{\alpha}, \tag{18}$$

where $d \equiv \frac{\rho(1-\alpha)}{1+\rho}$, $b_{im} \equiv \frac{cA_i}{\mu_m} = \frac{A_i}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}$ and $x^i_t \equiv \frac{w^i_t}{w^m_t} - d$. Note that $0 < d < 1/2$, and $b_{im} > 0$.

Let us analyze this non-linear first order difference equation. First note that because $0 < \alpha < 1$, this function is concave, and so we can have from 0 to 2 equilibria. Call $f(x^i_t) = -d + b_{im}\left(x^i_t\right)^{\alpha}$. The bifurcation point (where we pass from 0 to 2 equilibria), is defined by $f'(x^i_t) = 1$, and takes the values: $(x^i_t, x^i_{t+1}) = \left((\alpha b_{im})^{\frac{1}{1-\alpha}}, (\alpha b_{im})^{\frac{1}{1-\alpha}}\right)$. Therefore, the condition for existence of an equilibrium is $f\left((\alpha b_{im})^{\frac{1}{1-\alpha}}\right) \geq (\alpha b_{im})^{\frac{1}{1-\alpha}}$, or $-d + b_{im}(\alpha b_{im})^{\frac{\alpha}{1-\alpha}} \geq (\alpha b_{im})^{\frac{1}{1-\alpha}}$. In terms of parameters this condition is equivalent to: $\frac{A_m}{A_i} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$ for $i = r, p$.

Note that because $A_r > A_p$, it is enough to impose the condition $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$ to guarantee the existence of an equilibrium in our model (Condition 1). Note that for $\rho, \alpha \in (0, 1), 1 < \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. Therefore, for any value of ρ, α it will exist A_m and A_p such that $A_m > A_p$ and $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. Thus, for A_m sufficiently close to A_p we know that an equilibrium exists.

Assuming that this condition is satisfied with strict inequality, we will have two balanced growth equilibria, x^{i1} and x^{i2}, both defined by the solution to the equation $x^i = -d + b_{im}\left(x^i\right)^{\alpha}$. Because the function $f(\cdot)$ is concave, the first one, x^{i1}, is unstable and second one, x^{i2}, is stable (note that $f'(x^{i1}) > 1$ while $f'(x^{i2}) < 1$).

Convergence

It is easy to verify that if x^i_0 is at the right of x^{i1}, the economy will converge to x^{i2}. If the initial distribution of wealth implies a x^i_0 that is at the left of x^{i1}, then there is no convergence.

Note now that $x^r_0 \equiv \frac{w^r_0}{w^m_0} - d$ is always at the right of x^{r1}. To see this note first that $\frac{w^r_0}{w^m_0} > 1$, in which case $x^r_0 > 1 - d$. Now, if we can prove that $f(1-d) > 1-d$, then we will know that $1-d$ is at the right of x^{r1} (more precisely $x^{r1} < 1-d < x^{r2}$) and because $x^r_0 > 1-d$ then x^r_0 will be as well at the right of x^{r1} . It can be verified that $f(1-d) > 1-d$ if and only if $\frac{A_r}{A_m} > 1$,[17] and the later condition is true by definition.

[17] $f(1-d) > 1-d \iff -d + b_{rm}(1-d)^{\alpha} > 1-d \iff b_{rm}(1-d)^{\alpha} > 1 \iff \frac{A_r}{A_m}(\frac{1+\rho}{1+\rho\alpha})^{\alpha}(\frac{1+\rho\alpha}{1+\rho})^{\alpha} > 1 \iff \frac{A_r}{A_m} > 1$ QED.

Thus, $x_0^r > x^{r1}$ and the relative wealth of the rich class will always converge to the stable equilibrium x^{r2}.

Unfortunately the same cannot be said about x_0^p. Note that $0 < \frac{w_0^p}{w_0^m} < 1$, therefore x_0^p is restricted by definition to the interval $(-d, 1-d)$, i.e. $-d < x_0^p \equiv \frac{w_0^p}{w_0^m} - d < 1 - d$. To ensure convergence it is necessary to impose additionally the condition $x^{p1} < x_0^p$ (Condition 2). The question is, does such x_0^p exist? The answer is yes it does. We must prove that exist x_0^p such that $x^{p1} \le x_0^p < 1 - d$. In other words we must prove that $x^{p1} < 1 - d$ or that the interval $(x^{p1}, 1-d)$ is not empty.

First, it can be verified that $f(1-d) < 1 - d$ if $\frac{A_p}{A_m} < 1$ (and this is the case here), then $1 - d < x^{p1}$ or $1 - d > x^{p2}$. Second, the following inequality $(\alpha b_{pm})^{\frac{1}{1-\alpha}} < 1 - d$ is also always verified for $\frac{A_p}{A_m} < 1$,[18] and $x^{p1} < (\alpha b_{pm})^{\frac{1}{1-\alpha}}$ (since we are assuming $f\left((\alpha b_{pm})^{\frac{1}{1-\alpha}}\right) > (\alpha b_{pm})^{\frac{1}{1-\alpha}}$ and therefore $x^{p1} < (\alpha b_{pm})^{\frac{1}{1-\alpha}} < x^{p2}$), then $x^{p1} < 1 - d$ (in fact $x^{p1} < x^{p2} < 1 - d$). Therefore, the interval $(x^{p1}, 1-d)$ is not empty. QED

Growth rates of other variables

All the variables are linked to wealth. We will now prove that if wealth is growing at rate $\mu_m - 1$, then all the other variables of the economy will grow at the same rate.

Growth rate of g_t

Note that

$$g_t = g_t^m = \frac{\rho}{1+\rho}(1-\alpha)w_t^m. \tag{19}$$

Dividing this expression by the equation lagged one period, and using $\frac{w_t^m}{w_{t-1}^m} = \mu_m$, we have that

$$\frac{g_t}{g_{t-1}} = \frac{w_t^m}{w_{t-1}^m} = \mu_m. \tag{20}$$

18 $\frac{A_p}{A_m} < \frac{1/\alpha+\rho}{1+\rho} \Leftrightarrow \frac{A_p}{A_m} < \frac{1}{\alpha}\frac{1+\rho\alpha}{1+\rho}$
$\Leftrightarrow \frac{A_p}{A_m} < \frac{1}{\alpha}\left(\frac{1+\rho\alpha}{1+\rho}\right)^{1-\alpha}\left(\frac{1+\rho\alpha}{1+\rho}\right)^{\alpha} \Leftrightarrow \alpha\frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha} < \left(\frac{1+\rho\alpha}{1+\rho}\right)^{1-\alpha}$
$\Leftrightarrow (\alpha b_{pm})^{\frac{1}{1-\alpha}} = \left[\alpha\frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}} < \left(\frac{1+\rho\alpha}{1+\rho}\right) = 1 - d$. Note that the first inequality is verified for $\frac{A_p}{A_m} < 1$.

Growth rate of k_t^i for $i = p, m, r$

From (7) we have that

$$k_t^i = \frac{\rho\alpha}{1+\rho\alpha}(w_t^i - g_t). \tag{21}$$

Now, using the fact that $g_t = \mu_m g_{t-1}$ [Eq. (20)] and $w_t^i = \mu_m w_{t-1}^i$, the 1 + growth rate of the investment is

$$\frac{k_t^i}{k_{t-1}^i} = \frac{(w_t^i - g_t)}{(w_{t-1}^i - g_{t-1})} = \mu_m \frac{(w_{t-1}^i - g_{t-1})}{(w_{t-1}^i - g_{t-1})} = \mu_m. \tag{22}$$

Growth rate of y_t^i for $i = p, m, r$

Because the production functions have constant returns to scale with respect to k_t^i and g_t and both variables are growing at the same rate $\mu_m - 1$, y_t^i will grow at this rate.

Properties of the indirect utility functions

The indirect utility function for an individual of class i for $w_t^i > g_t$ is

$$v_t^i(w_t^i, g_t) = D_i + (1+\rho\alpha)\log(w_t^i - g_t) + \rho(1-\alpha)\log(g_t). \tag{23}$$

Because $D_p < D_m < D_r$ (since $A_p < A_m < A_r$) and $w_t^p < w_t^m < w_t^r$ we will have that $v_t^p(w_t^p, g_t) < v_t^m(w_t^m, g_t) < v_t^r(w_t^r, g_t)$, i.e. for a given level of g_t the utility is higher as higher is the wealth.

We already know that the indirect utility functions are concave and have a maximum at $g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i$, then $g_t^p < g_t^m < g_t^r$. Additionally, $\lim_{g_t \longrightarrow w_t^i} v_t^i(w_t^i, g_t) = -\infty$, and when $g_t = 0$, $u_t^i = \log(w_t^i)$ (since $k_t^i = y_t^i = 0$).

$\bar{g}_t^p < \bar{g}_t^m < \bar{g}_t^r$

We will prove that $\bar{g}_t^p < \bar{g}_t^m < \bar{g}_t^r$.

Totally differentiating $\log(w_t^i) = D_i + (1+\rho\alpha)\log(w_t^i - g_t) + \rho(1-\alpha)\log(g_t)$ we can find the effect of an increase in wealth on g_t^i (this is valid even when the increase is from w_t^p to w_t^m or to w_t^r because the indirect utility functions are identical, except for the term D_i, then we just have to take into account the additional increase due to changes in D_i when we change classes)

$$\frac{dg_t}{dw^i_t}_{g_t=\bar{g}^i_t} = \frac{\left(\frac{1+\rho\alpha}{w^i_t-\bar{g}^i_t} - \frac{1}{w^i_t}\right)}{\left(\frac{1+\rho\alpha}{w^i_t-\bar{g}^i_t} - \frac{\rho(1-\alpha)}{\bar{g}^i_t}\right)} > 0. \tag{24}$$

Note that the numerator is always positive since $1 + \rho\alpha > 1$ and $w^i_t - \bar{g}^i_t < w^i_t$, and the denominator is also positive because $\bar{g}^i_t > g^i_t = \frac{\rho(1-\alpha)}{1+\rho} w^i_t$ (note that at $g_t = g^i_t$ the denominator is zero, from there up, i.e. for $\bar{g}^i_t > g^i_t$, is positive). When we go from $i = p$ to $i = m$, i.e. from w^p_t to w^m_t, we have then that $\bar{g}^m_t > \bar{g}^p_t$ (as mentioned above the increase will be larger than (24) suggests because we have to add an additional positive effect due to the increase in D_i).

Proposition 5

Similar to Propositions 2 and 4.

$$w^p_t < w^m_t < w^r_t \quad \forall t.$$

See no social mobility property above.

Existence

Now the condition will be $\frac{A_p}{A_m} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. But note that this condition is always verified since $\frac{A_p}{A_m} < 1$ and $\frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha} > 1$ for $\rho, \alpha \in (0, 1)$.

Convergence

Note that both $\frac{w^m_0}{w^p_0}, \frac{w^r_0}{w^p_0} > 1$. Thus, following the same lines of reasoning that in the proof of Propositions 2 and 5 it can be shown that we always begin at the right of the unstable equilibrium. Therefore, there is always convergence to the stable equilibrium.

Growth rates of other variables

All variables are growing at rate $\mu_p - 1$. Proof: same as in Propositions 2 and 4.

Proposition 6

Similar to Propositions 2 and 4.

This case is the most demanding in terms of the conditions that are necessary to verify for existence and convergence to equilibrium.

$$w_t^p < w_t^m < w_t^r \quad \forall t.$$

See no social mobility property above.

Existence

Now the condition will be $\frac{A_r}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$.

Convergence

Note that both $\frac{w_0^p}{w_0^r}, \frac{w_0^m}{w_0^r} < 1$. Therefore for convergence we must impose the condition that both $\frac{w_0^p}{w_0^r} - d$ and $\frac{w_0^m}{w_0^r} - d$ are at the right of the unstable equilibrium.

Growth rates of other variables

All variables are growing at rate $\mu_r - 1$. Proof: same as in Propositions 2 and 4.

Alternative default policy

Assume that the middle class has plurality. For period 0 this class will be able to choose $g_0 = g_0^m$, because for the rich class g_0^m is better than the default policy $g_0 = 0$. It is easy to prove this, just note that because we are in the increasing region of the utility function, any public investment g_0 strictly greater than 0 and smaller than g_0^r is better than the default public policy $g_0 = 0$. Note that $0 < g_0^m < g_0^r$ then g_0^m is preferred to 0.

Next for $t = 1$, note that because we are assuming that the parameters are such that there is economic growth, i.e. $w_1^m > w_0^m$, then $g_1^m = \frac{\rho(1-\alpha)}{1+\rho} w_1^m > g_0^m = \frac{\rho(1-\alpha)}{1+\rho} w_0^m$. Additionally, by the no-social-mobility property $w_1^r > w_1^m$ (Appendix A), therefore $g_1^r = \frac{\rho(1-\alpha)}{1+\rho} w_1^r > g_1^m = \frac{\rho(1-\alpha)}{1+\rho} w_1^m$. Therefore $g_0^m < g_1^m < g_1^r$ and then for the same argument as before the rich class will prefer g_1^m to g_0^m.

Following the same steps as before we can prove that the rich class will prefer g_t^m to g_{t-1}^m as long as $g_{t-1}^m < g_t^m < g_t^r$ (and this is true by the no-social-mobility property).

By complete induction this will be true for any t.

Following the same reasoning as before, we can prove that if the poor class has plurality, then for both m and r is better to accept g_t^p in period t than the default policy g_{t-1}^p.

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